

# Optimal Decision Making in Operations Research and Statistics

*Methodologies and Applications*

*Editors*

**Irfan Ali, Leopoldo Eduardo Cárdenas-Barrón,  
Aquil Ahmed and Ali Akbar Shaikh**



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# Preface

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Operations Research (OR) has become a powerful technique for optimal decision-making. New techniques and sophisticated analysis tools are required to resolve the challenges arising from modern problems. It leads to the emergence of OR for efficiently determining optimal solutions to problems of real world. Although there are many types of conceivable problems, OR practitioners and researchers have found several problems in different circumstances. Thus, a challenge problem may be in the manufacturing industry area while another may be in the service sector. However, their essential features are the same. Thus, it is possible to describe these problems by naming the general categories into which they fall irrespective of their physical descriptions. A common analytical technique can be used to find the optimal solution to problems belonging to the same general category. In this direction, OR helps make better decision and solve problems in the real world. It uses mathematical relations, statistical computations, engineering techniques, economics and management methodologies to know the consequences of deciding for any possible alternative actions.

The decision-making techniques can be used in industries and services for making business decisions under risk and uncertainty. Furthermore, the decision-making techniques are also applied successfully to almost every possible sphere of human activity. Moreover, decision-making techniques are widely applied in different fields, ranging from almost every branch of science, engineering, industrial management, management planning, medical sciences, social sciences and economics, among others.

The book “Optimal Decision Making in Operations Research & Statistics: Methodologies and Applications” has been written by unified authors with a diverse background expertise from the faculties of Operations Research, Management, Applied Statistics and Mathematics. The contributed chapters are based on the vast research experiences of the authors in real-world decision-making problems.

The book is on the recent developments and contributions in optimal decision-making using optimization and statistical techniques. Mathematical modelling of cost-effective management policies are also part of the book.

The book presents challenging and practical real-world applications based on decision-making problems in various fields. The modelling and solution procedures of such real-world problems are provided concisely. This book provides readers a valuable compendium of several decision-making problems as a reference for this field’s researchers and industrial practitioners. After reading this book, the readers will understand the formulations of decision-making problems and their solution procedures using appropriate optimization and statistical techniques.

This book broadly covers applications of applied statistics and optimization techniques in decision making in the various areas such as—estimation, control charts, econometric, regression, sampling, stochastic modelling, inventory control and management, transportation problem and optimization.

Finally, this book benefits the teachers, students, researchers, and industrialists working in material science, especially Operations Research and Applied Statistics, as a valuable reference handbook for teaching, learning, and research.

# Contents

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|  |     |
|--|-----|
| <i>Preface</i>   | iii |
| 1. A New Version of the Generalized Rayleigh Distribution with Copula, Properties, Applications and Different Methods of Estimation<br><i>M Masoom Ali, Haitham M Yousof and Mohamed Ibrahim</i>       | 1   |
| 2. Expanding the Burr X Model: Properties, Copula, Real Data Modeling and Different Methods of Estimation<br><i>M Masoom Ali, Mohamed Ibrahim and Haitham M Yousof</i>                                 | 21  |
| 3. Transmuted Burr Type X Model with Applications to Life Time Data<br><i>Tabassum Naz Sindhu, Zawar Huassian and Muhammad Aslam</i>   | 43  |
| 4. Monitoring Patients Blood Level through Enhanced Control Chart<br><i>Muhammad Aslam, Khushnoor Khan and Nasrullah Khan</i>  | 59  |
| 5. Goodness of Fit in Parametric and Non-parametric Econometric Models<br><i>Shalabh, Subhra Sankar Dhar and N Balakrishna</i>   | 68  |
| 6. Stochastic Models for Cancer Progression and its Optimal Programming for Control with Chemotherapy<br><i>Tirupathi Rao Padi</i>   | 91  |
| 7. A New Unrelated Question Model with Two Questions Per Card<br><i>Tonghui Xu, Stephen A Sedory and Sarjinder Singh</i>   | 117 |
| 8. Hybrid of Simple Model and a New Unrelated Question Model for Two Sensitive Characteristics<br><i>Renhua Zheng, Stephen A Sedory and Sarjinder Singh</i>  | 127 |
| 9. Hybrid of Crossed Model and a New Unrelated Question Model for Two Sensitive Characteristics<br><i>Renhua Zheng, Stephen A Sedory and Sarjinder Singh</i>   | 165 |
| 10. Modified Regression Type Estimator by Ingeniously Utilizing Probabilities for more Efficient Results in Randomized Response Sampling<br><i>Roberto Arias, Stephen A Sedory and Sarjinder Singh</i> | 206 |
| 11. Ratio and Regression Type Estimators for a New Measure of Coefficient of Dispersion Relative to the Empirical Mode<br><i>Christin Variathu Eappen, Stephen A Sedory and Sarjinder Singh</i>        | 224 |
| 12. Class of Exponential Ratio Type Estimator for Population Mean in Adaptive Cluster Sampling<br><i>Akingbade Toluwalase Janet and Balogun Oluwafemi Samson</i>                                       | 272 |
| 13. An Inventory Model for Substitutable Deteriorating Products under Fuzzy and Cloud Fuzzy Demand Rate<br><i>Nita H Shah and Milan B Patel</i>  | 280 |
| 14. Co-ordinated Selling Price and Replenishment Policies for Duopoly Retailers under Quadratic Demand and Deteriorating Nature of Items<br><i>Nita H Shah and Monika K Naik</i>                       | 291 |
| 15. Quadratic Programming Approach for the Optimal Multi-objective Transportation Problem<br><i>Masar Al-Rabeeah, Ali Al-Hasani and M G M Khan</i>   | 302 |
| 16. Analyzing Multi-Objective Fixed-Charge Solid Transportation Problem under Rough and Fuzzy-Rough Environments<br><i>Sudipta Midya and Sankar Kumar Roy</i>  | 308 |

|  |     |
|--|-----|
| 17. Overall Shale Gas Water Management: A Neutrosophic Optimization Approach<br><i>Ahmad Yusuf Adhami, Firoz Ahmad and Nahida Wani</i>   | 321 |
| 18. Memory Effect on an EOQ Model with Price Dependant Demand and Deterioration<br><i>Mostafijur Rahaman, Sankar Prasad Mondal and Shariful Alam</i>                                       | 334 |
| 19. Optimality Conditions of an Unconstrained Imprecise Optimization Problem via Interval Order Relation<br><i>Md Sadikur Rahman and Asoke Kumar Bhunia</i>                                | 344 |
| 20. Power Comparison of Different Goodness of Fit Tests for Beta Generalized Weibull Distribution<br><i>Kanchan Jain, Neetu Singla and Suresh K Sharma</i>                                 | 352 |
| 21. On the Transmuted Modified Lindley Distribution: Theory and Applications to Lifetime Data<br><i>Lishamol Tomy, Christophe Chesneau and Jiju Gillariose</i>                             | 361 |
| 22. Adjusted Bias and Risk for Estimating Treatment Effect after Selection with an Application in Idiopathic Osteoporosis<br><i>Omer Abdalghani, Mohd Arshad, K R Meena and A K Pathak</i> | 370 |
| 23. Validity Judgement of an EOQ Model using Phi-coefficient<br><i>Suman Maity, Sujit Kumar De, Madhumangal Pal and Sankar Prasad Mondal</i>   | 378 |
| 24. Uncertain Chance-Constrained Multi-Objective Geometric Programming Problem<br><i>Sahidul Islam</i>   | 388 |
| 25. Optimal Decision Making for the Prediction of Diabetic Retinopathy in Type 2 Diabetes Mellitus Patients<br><i>Faiz Noor Khan Yusufi, Nausheen Hashmi, Aquil Ahmed and Jamal Ahmad</i>  | 406 |
| <i>Index</i>   | 425 |

## CHAPTER 21

# On the Transmuted Modified Lindley Distribution Theory and Applications to Lifetime Data

*Lishamol Tomy,<sup>1</sup> Christophe Chesneau<sup>2,\*</sup> and Jiju Gillarirose<sup>3</sup>*

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### 1. Introduction

In many applied sciences, modeling and analyzing lifetime data using lifetime distributions have received the attention of several researchers. Undoubtedly, the one-parameter Lindley distribution (Lindley, 1958, 1965) is one of the most attractive distributions in Statistics. There are many extensions of the Lindley distribution to provide flexibility for modeling data. In particular, see, Zakerzadeh and Dolati, 2009; Nadarajah et al., 2011; Shanker and Mishra, 2013a; Ghitany et al., 2013; Singh et al., 2014; Shanker and Mishra, 2013b; Sharma et al., 2015; Sharma et al., 2016. The interested reader can find a comprehensive review on the Lindley distribution in Tomy 2018. More recently, a new modified Lindley (ML) distribution has been proposed by Chesneau et al., 2020a as a simple one-parameter alternative to the exponential and Lindley distributions. It is defined with the following survival function:

$$G(x) = \left[ 1 + \frac{\theta x}{1 + \theta} e^{-\theta x} \right] e^{-\theta x}, \quad x > 0, \quad (1)$$

with  $\theta > 0$ . An important property of the ML distribution is that its probability density function (pdf) can be expressed as a linear combination of exponential and gamma pdfs. In practical contexts, the ML distribution is a strong one-parameter competitor to the Lindley and exponential distributions. In addition to this, Chesneau et al., 2020b; Chesneau et al., 2020c further studied two extensions for the ML distribution, such as the inverse ML and wrapped ML distributions, respectively, and presented their statistical properties.

In the last decades, several researchers have added new parameters to expanding classical distributions in order to improve the modeling of survival data. In this regard, numerous meaningful new families of distributions have been built towards the generalization of well-established classical lifetime distributions. The transmuted family was first proposed by Shaw and Buckley, 2007 based on the transmutation method and the theory was further clearly mentioned in Shaw and Buckley, 2009. The transmuted generated (T-G) family of distributions is characterized by the cumulative density function (cdf) given by

$$F(x) = G(x)[1 + \beta - \beta G(x)], \quad x \in \mathbb{R}, \quad (2)$$

where  $\beta \in [-1, 1]$ ,  $\beta$  introduces the skewness and varies the corresponding tail weights, and  $G(x)$  denotes the cdf of a parent continuous distribution. For more details about the quadratic rank transmutation map, see Shaw and Buckley, 2009. Subsequently, the T-G transformation was applied to several well-known distributions from its inception. For example, Aryal and Tsokos, 2009; Aryal and Tsokos, 2011 derived the two transmuted transformed distributions such as, transmuted extreme value and transmuted Weibull distributions. Aryal, 2013 suggested the transmuted log-logistic distribution and its various properties. Merovci, 2013a introduced the transmuted Lindley distribution and applied it to bladder cancer data. Merovci, 2013b proposed the transmuted exponentiated exponential distribution. The transmuted Lindley-geometric distribution has been discussed by Merovci and Elbatal, 2013. Ashour and Eltehiwy, 2013a deduced the transmuted Lomax distribution and Ashour and Eltehiwy, 2013b created the transmuted exponentiated Lomax distribution. Eltehiwy and Ashour, 2013 developed transmuted exponentiated modified Weibull distribution. The transmuted exponentiated gamma distribution has been discussed by Hussian, 2014. Elbatal et al., 2014 suggested various estimation methods

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